Francisco Safara: 52856

Ciências ULisboa

Linear Perturbation Theory in GR

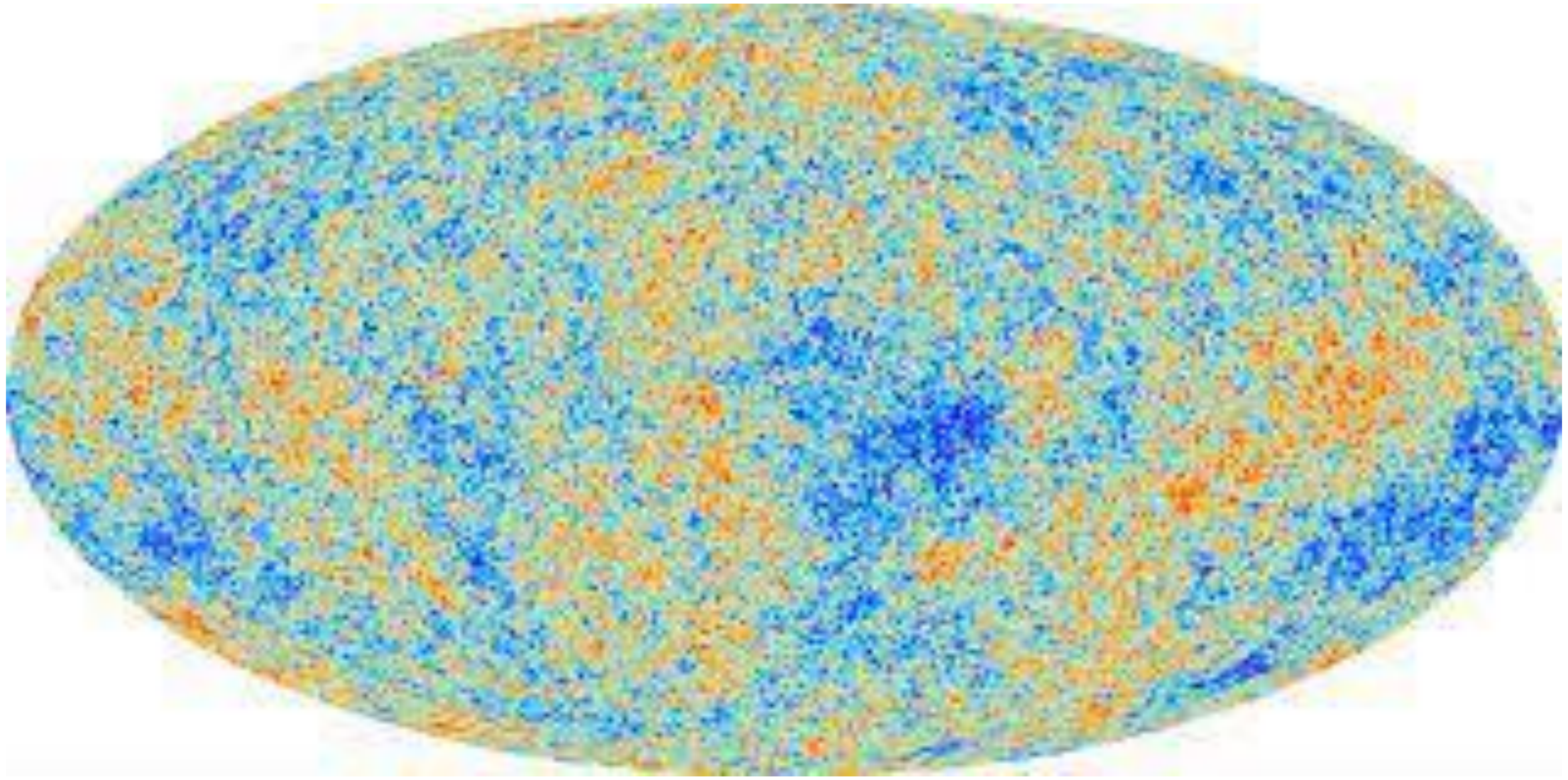
Primordial Universe

01

Introduction

- The universe is/was, on the largest scales, nearly perfectly isotropic and homogeneous.

- However, when we look at the night sky we see a huge diversity of structures: stars, galaxies, and clusters of galaxies and beyond.



02 Introduction

- The theory of cosmological perturbations is what allows us to connect theories of the very early Universe with the data on the large-scale structure of the Universe at late times and is thus of central importance in modern cosmology.

- Cosmological Perturbations:

  - ❑ Newtonian perturbation theory;
  - ❑ General relativity perturbations theory;

03 Newtonian perturbation theory

- We model the universe as a fluid where the fluid`s evolution is governed by the standard fluid dynamics equations:

$$\partial_0 \rho + \nabla_r \cdot (\rho \mathbf{u}) = 0,$$ → continuity equation

$$(\partial_0 + \mathbf{u} \cdot \nabla_\mathbf{r})\mathbf{u} + \rho^{-1} \nabla_\mathbf{r} P + \nabla_\mathbf{r} \Phi = 0,$$ → Euler equation

$$\nabla_\mathbf{r}^2 \Phi = 4\pi G \rho,$$ → poison equation

04 Newtonian perturbation theory

- The Newtonian perturbation approach allows us to study what these equations imply for the evolution of small perturbations around a homogeneous background:

$$\rho = \bar{\rho} + \delta\rho, \qquad P = \bar{P} + \delta P, \qquad \Phi = \bar{\Phi} + \delta\Phi, \qquad \mathbf{u} = \bar{\mathbf{u}} + \delta\mathbf{u}.$$

- For an expanding fluid:

$$[\partial_0 - H\mathbf{x}\cdot\nabla_{\mathbf{x}}][\bar{\rho} + \delta\rho] + a^{-1}\nabla_{\mathbf{x}}\cdot[(\bar{\rho} + \delta\rho)(Ha\mathbf{x} + \mathbf{v})] = 0,$$

$$\partial_0\mathbf{v} + H\mathbf{v} + (a\bar{\rho})^{-1}\nabla_{\mathbf{x}}\delta P + a^{-1}\nabla_{\mathbf{x}}\delta\Phi = 0,$$

$$\nabla^2_{\mathbf{x}}\delta\Phi = 4\pi G a^2 \delta\rho.$$

proper coordinates → $\mathbf{r}$; "comoving" coordinates → $\mathbf{x}$

$$\mathbf{r} = a\mathbf{x},$$

$$\frac{d\mathbf{r}}{dt} \equiv \mathbf{u} = H\mathbf{r} + \mathbf{v},$$

$$H = \frac{\dot{a}}{a}$$

$$\mathbf{v} = a\frac{d\mathbf{x}}{dt}$$

05 General relativity perturbations theory-introduction

- The basic idea is to consider a small perturbation to the background metric, giving:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

flat FRLW metric

- The full metric may be written in conformal time as:

$$ds^2 = a^2(\eta)\left[(1+2A)d\eta^2 - 2B_i d\eta dx^i - \{\delta_{ij} + h_{ij}\}\, dx^i dx^j\right]$$

- There are, then, at first sight, 10 fluctuations degrees of freedom in (not 16 because of the symmetry)

$$dt = a(t)d\eta$$

06 General relativity perturbations theory-introduction

$$ds^2 = a^2(\eta)\left[(1+2A)d\eta^2 - 2B_i d\eta dx^i - \{\delta_{ij}+h_{ij}\}\, dx^i dx^j\right]$$

- We may decompose the perturbations in scalar, vectors and tensor members (SVT decomposition):

$$B_i = \partial_i B + \hat{B}_i$$

$$h_{ij} = 2C\delta_{ij} + 2\left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)E + (\partial_i\hat{E}_j + \partial_j\hat{E}_i) + 2E_{ij}.$$

where vector quantities are divergence free and tensor quantities are transverse and traceless.

07 General relativity perturbations theory-introduction

- The total number of degrees of freedom may be decreased by simply discarding the contributions of the vector and tensor perturbation members. This can be done because:
  - ❑ the most important fluctuations, at least in inflationary cosmology, are the scalar metric fluctuations;
  - ❑ in linear theory there is no coupling between the different fluctuation modes, and hence they evolve independently;
- The scalar fluctuations are the fluctuations which couple to matter inhomogeneities and which **are the relativistic generalization of the Newtonian perturbations.**
- The perturbed metric can now be written as:

$$ds^2 = a^2(\eta)\left[(1+2A)d\eta^2 - 2(\partial_i B)d\eta dx^i - \left\{(1+2C)\delta_{ij} + 2(\partial_i\partial_j - \delta_{ij}\frac{1}{3}\nabla^2)E\right\}dx^i dx^j\right]$$

08 General relativity perturbations theory - gauge problem

- The metric perturbations aren't uniquely defined but depend on our choice of coordinates or the gauge choice. By performing a small-amplitude transformation (gauge transformation) of the space-time coordinates we can easily introduce "fictitious" fluctuations.
- We need, then, a more physical way to identify true perturbations.
- How does the coordinate transformations act on our metric?

$$X^\mu \longrightarrow \tilde{X}^\mu \equiv X^\mu + \xi^\mu \quad \text{where} \quad \begin{aligned} \xi^0 &\equiv T \\ \xi^i &\equiv \partial^i L + \hat{L}^i \end{aligned}$$

- By using the invariance of the space time interval:

$$g_{\mu\nu}(X) = \tilde{g}_{\alpha\beta}(\tilde{X}) \frac{\partial \tilde{X}^\alpha}{\partial X^\mu} \frac{\partial \tilde{X}^\beta}{\partial X^\nu}$$

09 General relativity perturbations theory - gauge problem

- We obtain, then:

derivative in conformal time

$$A \longrightarrow \tilde{A} = A - T' - \mathcal{H}T \qquad B \longrightarrow B + T - L' \qquad C \longrightarrow C - \mathcal{H}T - \frac{1}{3}\nabla^2 L \qquad E \longrightarrow E - L$$

$$\mathcal{H} = \frac{a'}{a}$$

- As an example:

$$g_{00}(X) = \tilde{g}_{00}(\tilde{X}) \left(\frac{\partial\tilde{\eta}}{\partial\eta}\right)^2 \longrightarrow$$

$$\begin{aligned}
[1+2A]a^2(\eta) &= [1+2\tilde{A}]a^2(\eta+T)\left[\frac{\partial(\eta+T)}{\partial\eta}\right]^2 = \\
&= [1+2\tilde{A}][a(\eta)+a'(\eta)T+\cdots]^2[1+T']^2 = \\
&= [1+2\tilde{A}][a(\eta)+a'(\eta)T+\cdots]^2[1+2T'+T'^2] = \\
&= [1+2\tilde{A}+2T'][a^2(\eta)+2a(\eta)a'(\eta)T] = \\
&= [1+2\tilde{A}+2T'+2\mathcal{H}T]a^2(\eta)
\end{aligned}$$

$$ds^2 = a^2(\eta)\left[(1+2A)d\eta^2 - 2(\partial_i B)d\eta dx^i - \left\{(1+2C)\delta_{ij} + 2(\partial_i\partial_j - \delta_{ij}\frac{1}{3}\nabla^2)E\right\}dx^i dx^j\right]$$

$$dt = a(t)d\eta$$

10 General relativity perturbations theory - gauge problem

- One way to avoid the gauge problem is to define special combinations of metric perturbations that do not transform under a change of coordinates. Here we mention the *Bardeen variables* :

$$\Psi \equiv A + \mathcal{H}(B - E') + (B - E')'$$

$$\Phi \equiv -C + \mathcal{H}(B - E') + \frac{1}{3}\nabla^2 E$$

- As an example:

$$\begin{aligned}\Psi \longrightarrow \tilde{\Psi} &= \tilde{A} + \mathcal{H}(\tilde{B} - \tilde{E}') + (\tilde{B} - \tilde{E}')' = \\ &= A - T' - \mathcal{H}T + \mathcal{H}(B + T - L' - [E - L]') + (B + T - L' - [E - L]')' = \\ &= A - T' + \mathcal{H}(B - E') + (B' + T' - E'') = \\ &= A + \mathcal{H}(B - E') + (B - E')' = \Psi\end{aligned}$$

General relativity perturbations theory - gauge problem

- We can use the freedom in the gauge functions to set two of the four scalar metric perturbations to zero.
- One useful choice is the *Newtonian gauge* :

$$B = E = 0$$

$$\downarrow$$

$$ds^2 = a^2(\eta)\left[(1+2\Psi)d\eta^2 - (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$

$$A \equiv \Psi \qquad\qquad C \equiv -\Phi$$

12 General relativity perturbations theory - Evolution Equations

$$ds^2 = a^2(\eta)\left[(1+2\Psi)d\eta^2 - (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$

$\big\downarrow$ final goal

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

13 General relativity perturbations theory - Evolution Equations

- We begin by remembering the definition of the Christopher symbols:

$$\Gamma^{\mu}_{\nu\rho} = \frac{1}{2} g^{\mu\lambda} \left(\partial_\nu g_{\lambda\rho} + \partial_\rho g_{\lambda\nu} - \partial_\lambda g_{\nu\rho} \right)$$

- As an example:

$$\begin{aligned}
\Gamma^0_{00} &= \frac{1}{2} g^{00} \left(\partial_0 g_{00} + \partial_0 g_{00} - \partial_0 g_{00} \right) = \frac{1}{2} g^{00} \partial_0 g_{00} = \\
&= \frac{1}{2} \left[\frac{1 - 2\Psi}{a^2} \right] \partial_0 \left[a^2 (1 + 2\Psi) \right] = \\
&= \frac{1}{2} \left[\frac{1 - 2\Psi}{a^2} \right] \left[2a^2 \Psi' + 2aa' (1 + 2\Psi) \right] = \\
&= [1 - 2\Psi][\Psi' + \mathcal{H}(1 + 2\Psi)] = \\
&= \Psi' + \mathcal{H}(1 + 2\Psi) - 2\mathcal{H}\Psi = \\
&= \mathcal{H} + \Psi'
\end{aligned}$$

$$g^{\mu\nu} = a^{-2} \begin{bmatrix} 1 - 2\Psi & 0 \\ 0 & -(1 + 2\Phi)\delta^{ij} \end{bmatrix}$$

$$ds^2 = a^2(\eta) \left[(1 + 2\Psi) d\eta^2 - (1 - 2\Phi) \delta_{ij} dx^i dx^j \right]$$

14 General relativity perturbations theory - Evolution Equations

- With an analogous logic we arrive at:

$$\Gamma^0_{0i} = \partial_i \Psi$$

$$\Gamma^i_{00} = \delta^{ij}\partial_j \Psi$$

$$\Gamma^0_{ij} = (\mathcal{H} - [\Phi' + 2\mathcal{H}(\Phi + \Psi)])\delta_{ij}$$

$$\Gamma^i_{j0} = (\mathcal{H} - \Phi')\delta^i_j$$

$$\Gamma^i_{jk} = [\delta^{il}\delta_{jk}\partial_l - (\delta^i_j\partial_k + \delta^i_k\partial_j)]\Phi$$

$$ds^2 = a^2(\eta)\left[(1+2\Psi)d\eta^2 - (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$

15 General relativity perturbations theory - Evolution Equations

- By knowing the Christopher symbols, we may, now, calculate the Ricci Tensor and Ricci scalar.
- The Ricci tensor can be defined as:

$$R_{\mu\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\mu\lambda} + \Gamma^\lambda_{\lambda\rho}\Gamma^\rho_{\mu\nu} - \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\rho}$$

- As an example:

$$\begin{aligned}
R_{00} &= \partial_\lambda \Gamma^\lambda_{00} - \partial_0 \Gamma^\lambda_{0\lambda} + \Gamma^\lambda_{\lambda\rho}\Gamma^\rho_{00} - \Gamma^\rho_{0\lambda}\Gamma^\lambda_{0\rho} = \\
&= \partial_i \Gamma^i_{00} - \partial_0 \Gamma^i_{0i} + \Gamma^i_{i\rho}\Gamma^\rho_{00} - \Gamma^\rho_{0i}\Gamma^i_{0\rho} = \\
&= \partial_i \Gamma^i_{00} - \partial_0 \Gamma^i_{0i} + [\Gamma^i_{i0}\Gamma^0_{00} + \Gamma^i_{ij}\Gamma^j_{00}] - [\Gamma^0_{0i}\Gamma^i_{00} + \Gamma^j_{0i}\Gamma^i_{0j}] = \\
&= \partial_i(\delta^{ij}\partial_j\Psi) - \partial_0((\mathcal{H} - \Phi')\delta^i_i) + [((\mathcal{H} - \Phi')\delta^i_i)(\mathcal{H} + \Psi') + 0] - [0 + (\mathcal{H} - \Phi')^2\delta^i_j\delta^j_i] = \\
&= \nabla^2\Psi - 3\partial_0(\mathcal{H} - \Phi') + 3(\mathcal{H} - \Phi')(\mathcal{H} + \Psi') - 3(\mathcal{H} - \Phi')^2 = \\
&= \nabla^2\Psi - 3(\mathcal{H}' - \Phi'') + 3(\mathcal{H}^2 - \mathcal{H}\Phi' + \mathcal{H}\Psi') - 3(\mathcal{H}^2 - 2\mathcal{H}\Phi') = \\
&= \nabla^2\Psi - 3(\mathcal{H}' - \Phi'') + 3\mathcal{H}(\Psi' + \Phi')
\end{aligned}$$

16 General relativity perturbations theory - Evolution Equations

- With an analogous logic we arrive at:

$$R_{0i} = 2\partial_i\Phi' + 2\mathcal{H}\partial_i\Psi$$

$$R_{ij} = [\mathcal{H}' + 2\mathcal{H}^2 - \Phi'' + \nabla^2\Phi - 2(\mathcal{H}' + 2\mathcal{H}^2)(\Phi + \Psi) - \mathcal{H}\Psi' - 5\mathcal{H}\Phi']\delta_{ij} + \partial_i\partial_j(\Phi - \Psi)$$

- The Ricci scalar may also be calculated:

$$\begin{aligned} R \equiv g^{\mu\nu}R_{\mu\nu} &= g^{00}R_{00} + 2g^{0i}R_{0i} + g^{ij}R_{ij} = \\ &= g^{00}R_{00} + g^{ij}R_{ij} = \\ &= a^{-2}[-6(\mathcal{H}' + \mathcal{H}^2) + 2\nabla^2\Psi - 4\nabla^2\Phi + 12(\mathcal{H}' + \mathcal{H}^2)\Psi + 6\Phi'' + 6\mathcal{H}(\Psi' + 3\Phi')] \end{aligned}$$

17 General relativity perturbations theory - Evolution Equations

- We are now finally in conditions to calculate the Einstein tensor, defined as:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

- We obtained:

$$G_{00} = R_{00} - \frac{1}{2} g_{00} R = 3\mathcal{H}^2 + 2\nabla^2\Phi - 6\mathcal{H}\Phi'$$

$$G_{0i} = R_{0i} - \frac{1}{2} g_{0i} R = R_{0i} = 2\partial_i\Phi' + 2\mathcal{H}\partial_i\Psi$$

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R = -(2\mathcal{H}' + \mathcal{H}^2)\delta_{ij} + [\nabla^2(\Psi - \Phi) + 2\Phi'' + 2(2\mathcal{H}' + \mathcal{H}^2)(\Psi + \Phi) + 2\mathcal{H}\Psi' + 4\mathcal{H}\Phi']\delta_{ij} + \partial_i\partial_j(\Phi - \Psi)$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

General relativity perturbations theory - Evolution Equations

- The Stress-Energy tensor in an homogeneous and isotropic universe gives the background Stress-Energy tensor:

$$\bar{T}^{\mu}_{\nu} = (\bar{\rho} + \bar{P})\bar{U}^{\mu}\bar{U}_{\nu} - \bar{P}\delta^{\mu}_{\nu}$$

- If we consider small perturbations of the Stress-Energy tensor:

$$T^{\mu}_{\nu} = \bar{T}^{\mu}_{\nu} + \delta T^{\mu}_{\nu}$$

where

$$\delta T^{\mu}_{\nu} = (\delta\rho + \delta P)\bar{U}^{\mu}\bar{U}_{\nu} + (\bar{\rho} + \bar{P})(\delta U^{\mu}\bar{U}_{\nu} + \bar{U}^{\mu}\delta U_{\nu}) - \delta P\delta^{\mu}_{\nu} - \Pi^{\mu}_{\nu}$$

General relativity perturbations theory - Evolution Equations

$$\bar{U}^\mu = a^{-1}(1, \mathbf{0}) \longrightarrow U^\mu = a^{-1}[1 - A, v^i]$$

$$\bar{g}_{\mu\nu}\bar{U}^\mu\bar{U}^\nu = 1$$

$$g_{\mu\nu}U^\mu U^\nu = 1$$

$$v^i = \frac{dx^i}{d\eta}$$

- Considering, again, just scalar perturbations:

$$\Pi_{ij} = (\partial_i\partial_j - \frac{1}{3}\nabla^2\delta_{ij})\Pi$$

$$v_i = \partial_i v$$

20 General relativity perturbations theory - Evolution Equations

- Using these results and the perturbed metric we can, after some calculations, obtain:

$$\delta T^0_0 = \delta\rho$$

$$\delta T^0_i = -(\bar{\rho} + \bar{P})\partial_i(v + \cancelto{0}{B})$$

$$\delta T^i_0 = (\bar{\rho} + \bar{P})\partial^i v \equiv \partial^i q$$

$$\delta T^i_j = -\delta P \delta^i_j - (\partial^i \partial_j - \frac{1}{3}\nabla^2 \delta^i_j)\Pi$$

21 General relativity perturbations theory - Evolution Equations

- Before we finally arrive at the Einstein equations we make a small detour to study the perturbed conservation equations from the relation:

$$\nabla_\mu T^\mu_\nu = \partial_\mu T^\mu_\nu + \Gamma^\mu_{\mu\alpha} T^\alpha_\nu - \Gamma^\alpha_{\mu\nu} T^\mu_\alpha = 0.$$

$\nu = 0$

$$\delta\rho' = -3\mathcal{H}(\delta\rho + \delta P) + 3\Phi'(\bar{\rho} + \bar{P}) - (\bar{\rho} + \bar{P})\nabla \cdot \mathbf{v},$$

$\nu = i$

$$\mathbf{v}' + \mathcal{H}\mathbf{v} - 3\mathcal{H}\mathbf{v}\frac{\bar{P}'}{\bar{\rho}'} + \frac{\nabla\delta P}{\bar{P} + \bar{\rho}} + \nabla\Psi = 0,$$

22 General relativity perturbations theory - Evolution Equations

- We are now finally in conditions to study the Einstein Equations:

$$G_{ij} = 8\pi G T_{ij} \quad (i \neq j) \quad \xrightarrow{*} \quad \partial_i \partial_j (\Phi - \Psi) = 0 \Rightarrow \Phi = \Psi$$

$$G_{ij} = -(2\mathcal{H}' + \mathcal{H}^2)\delta_{ij} + [\nabla^2(\Psi - \Phi) + 2\Phi'' + 2(2\mathcal{H}' + \mathcal{H}^2)(\Psi + \Phi) + 2\mathcal{H}\Psi' + 4\mathcal{H}\Phi']\delta_{ij} + \partial_i\partial_j(\Phi - \Psi)$$

$$\delta T^i_j = -\delta P \delta^i_j - (\partial^i \partial_j - \frac{1}{3}\nabla^2 \delta^i_j)\Pi$$

$$G_{00} = 8\pi G T_{00} \quad \longrightarrow \quad 3\mathcal{H}^2 + 2\nabla^2\Psi - 6\mathcal{H}\Psi' = 8\pi G a^2[\bar{\rho} + 2\Psi\bar{\rho} + \delta\rho]$$

$$G_{00} = 3\mathcal{H}^2 + 2\nabla^2\Phi - 6\mathcal{H}\Phi'$$

$$T_{00} = g_{0\mu} T^\mu_0 = g_{00} T^0_0 + g_{0i} T^i_0 = a^2(1 + 2\Psi)\{\bar{\rho} + \delta\rho\}$$

Friedman equation

$$3\mathcal{H}^2 = 8\pi G a^2 \bar{\rho} \Leftrightarrow \mathcal{H}^2 = \frac{8\pi G}{3} a^2 \bar{\rho}$$

$$2\nabla^2\Psi - 6\mathcal{H}\Psi' = 8\pi G a^2[2\Psi\bar{\rho} + \delta\rho]$$
$$\Leftrightarrow \nabla^2\Psi = 3\mathcal{H}\Psi' + 4\pi G a^2[2\Psi\bar{\rho} + \delta\rho]$$
$$\Leftrightarrow \nabla^2\Psi = 3\mathcal{H}(\Psi' + \mathcal{H}\Psi) + 4\pi G a^2 \delta\rho$$

*for simplicity is common to drop the anisotropic stress as this term usually does not play a significant role

23 General relativity perturbations theory - Evolution Equations

$$G_{0i} = 8\pi G T_{0i} \longrightarrow 2\partial_i\Psi' + 2\mathcal{H}\partial_i\Psi = -8\pi G a^2(\bar{\rho} + \bar{P})\partial_i v$$

$$G_{0i} = 2\partial_i\Phi' + 2\mathcal{H}\partial_i\Psi$$

$$T_{0i} = g_{0\mu}T^{\mu}_i = g_{00}T^0_i = \{a^2(1+2\Psi)\}\{-(\bar{\rho}+\bar{P})\partial_i v\}$$

$$\Psi' + \mathcal{H}\Psi = -4\pi G a^2(\bar{\rho}+\bar{P})v$$

Poison equation

$$\nabla^2\Psi = 3\mathcal{H}(-4\pi G a^2(\bar{\rho}+\bar{P})v) + 4\pi G a^2\delta\rho =$$
$$= 4\pi G a^2[-3\mathcal{H}(\bar{\rho}+\bar{P})v + \delta\rho] \equiv$$
$$\equiv 4\pi G a^2\bar{\rho}\Delta$$

$$G_{ii} = 8\pi G T_{ii} \longrightarrow -(2\mathcal{H}' + \mathcal{H}^2) + 2\Psi'' + 4(2\mathcal{H}' + \mathcal{H}^2)\Psi + 6\mathcal{H}\Psi' = 8\pi G a^2(\bar{P} + \delta P - 2\Psi\bar{P})$$

$$G_{ij} = -(2\mathcal{H}' + \mathcal{H}^2)\delta_{ij} + [\nabla^2(\Psi-\Phi) + 2\Phi'' + 2(2\mathcal{H}'+\mathcal{H}^2)(\Psi+\Phi) + 2\mathcal{H}\Psi' + 4\mathcal{H}\Phi']\delta_{ij} + \partial_i\partial_j(\Phi-\Psi)$$

$$T_{ii} = g_{i\mu}T^{\mu}_i = g_{ij}T^j_i = \{-(1-2\Psi)a^2\delta_{ij}\}\{-(\bar{P}+\delta P)\delta^j_i\}$$

acceleration equation

$$2\mathcal{H}' + \mathcal{H}^2 = -8\pi G a^2\bar{P}$$

$$2\Psi'' + 4(2\mathcal{H}' + \mathcal{H}^2)\Psi + 6\mathcal{H}\Psi' = 8\pi G a^2(\delta P - 2\Psi\bar{P})$$
$$\Leftrightarrow \Psi'' + 2(2\mathcal{H}' + \mathcal{H}^2)\Psi + 3\mathcal{H}\Psi' = 4\pi G a^2\delta P - 8\pi G a^2\Psi\bar{P}$$
$$\Leftrightarrow \Psi'' + 2(2\mathcal{H}' + \mathcal{H}^2)\Psi + 3\mathcal{H}\Psi' = 4\pi G a^2\delta P + (2\mathcal{H}' + \mathcal{H}^2)\Psi$$
$$\Leftrightarrow \Psi'' + (2\mathcal{H}' + \mathcal{H}^2)\Psi + 3\mathcal{H}\Psi' = 4\pi G a^2\delta P$$

Conclusion

- We derived the evolution equations for all matter and metric perturbations. In principle, we could now solve these equations. That study allows us to understand how the universe formed the astronomic structures and how small imperfections in the early universe have had great importance in its evolution. These results help us understand the universe, its constituents and its evolution.

QUESTIONS?